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Photon structure functions beyond the SUSY threshold

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Abstract. We evolve virtual photon parton densities up to the SUSY threshold and higher using coupled inhomogeneous DGLAP differential equations. Reliable input parameterizations were available from the c-quark threshold. Limited P^2 (target photon virtuality) dependence is observed. The difference to the photon structure function is shown to be significant with the introduction of SUSY dependent splitting functions. A negligible difference is observed by letting the gluino mass enter after the squark mass. An effort is made to include the squark threshold effect in such a way that both the renormalization group equations are satisfied and the perturbative calculation is reproduced.

1 Introduction

There has recently been a great deal of interest in the structure function of the photon. This is obtained from the scattering cross-section between a highly virtual photon with large square momentum Q^2 used as a probe and a nearly on-shell target photon with square momentum P^2, ($P^2 \ll Q^2$). If the target square momentum is also large (whilst maintaining the inequality $P^2 \ll Q^2$), the entire structure function can be calculated using renormalization group improved perturbation theory, whereas for low P^2 one is limited to a determination of the Q^2 dependence and, as in the case of deep-inelastic electron-proton scattering, one needs input information on the structure functions at some (low) value of Q^2, which cannot be determined by perturbation theory. A study of the photon structure function as a function of P^2 therefore provides information on the extrapolation between the perturbative and non-perturbative regimes of QCD.

Heuristically, one talks about separating the structure function into "direct" and "resolved" contributions. The former being exactly calculable in perturbation theory and the latter involving the uncalculable probability that the photon splits into other fundamental particles before being probed. Whereas such a picture is useful at the leading order level, higher order corrections mix these contributions. A formal and more precise analysis was first proposed by Witten [1] who pointed out that in an operator product expansion for photon-photon scattering the set of operators used in the case of photon-proton scattering must be augmented by a tower of photonic operators, whose matrix elements with the target photon are of order unity. In the (more intuitive) DGLAP approach one argues that since the probability of finding a particle other than a photon inside a photon is of order α_{em}, the probability of finding a "photon inside a photon" is unity plus corrections of order α_{em}. The DGLAP analysis must then be augmented by further off-diagonal splitting functions K_q and K_G which are interpreted as the perturbative probability for a photon to emit quark or gluon with a given fraction of its longitudinal momentum.

Interest in photon structure functions has recently been rekindled by the prospect of a future high-energy electron-positron collider with centre-of-mass energy of up to 1 TeV. Such a machine would enable an investigation of the photon structure function over a sufficiently wide range of Q^2 and P^2 to provide a stringent test of the evolution of these structure functions. Moreover, if the postulated existence of Supersymmetry (SUSY) turns out to be vindicated, these structure functions will reflect the existence of supersymmetric partners within photons. The contribution to the structure function due to the crossing of the threshold for the production of squarks, was first calculated by Reya [7]. However, a consistent analysis of the effect of supersymmetry on the photon structure function requires a full analysis of the enlarged DGLAP formalism in which above the SUSY threshold the standard splitting functions are augmented with splitting functions involving squarks and gluinos. This paper reports on such an analysis and displays results in which it can be seen that SUSY gives rise to a measurable increase in the Q^2 evolution of the structure photon structure function above threshold. Care must be taken to ensure a consistent treatment of the threshold behaviour for squark production as one passes through the threshold and this is discussed in detail.

The outline of the paper is as follows: In Sect. 2 we discuss the formalism, outlining the extension of the evolution equations to the regime in which squarks and gluinos are excited. We also give a description of the threshold treatment. Section 3 displays our results obtained from numerical solution of the enlarged evolution equations. We show the dependences on the SUSY threshold and on the square momentum P^2 of the target photon as well as on the usual variables Q^2 and Bjorken-x. In Sect. 5 we discuss our conclusions.

2 Formalism

We follow the formalism of Glück and Reya [2]. We will initially be concerned with quark and gluon distributions up to the SUSY threshold.

The nonsinglet (T_l) and singlet (Σ) sectors are treated separately,

$$\begin{aligned} T_3 &= 2(q_u - q_d) \\ T_8 &= 2(q_u + q_d - 2q_s) \\ T_{15} &= 2(q_u + q_d + q_s - 3q_c) \\ T_{24} &= 2(q_u + q_d + q_s + q_c - 4q_b) \\ T_{35} &= 2(q_u + q_d + q_s + q_c + q_b - 5q_t) \\ \Sigma &= 2\sum_i^f q_i \end{aligned}$$

where q_u, q_d, q_s, q_c, q_b and q_t refer to the relevant quark distributions. The factor of 2 accounts for the anti-quark distribution since for a photon $q_i = \bar{q}_i$. f runs over all relevant quark flavours. Each quark distribution is zero at and below its threshold hence each new non-singlet (T_l) is equal to the singlet (Σ) at threshold.

The evolution is controlled by the following inhomogeneous DGLAP differential equations,

$$\frac{dT_l}{d\,\ln Q^2} = P_{TT} \otimes T_l + K_T \tag{2.1}$$

for each singlet (T_l) and the coupled equations,

$$\frac{d\Sigma}{d\,\ln Q^2} = P_{\Sigma\Sigma} \otimes \Sigma + P_{\Sigma G} \otimes G + K_\Sigma$$

$$\frac{dG}{d\,\ln Q^2} = P_{G\Sigma} \otimes \Sigma + P_{GG} \otimes G + K_G \tag{2.2}$$

for the singlet (Σ) and gluon (G) sector.

For each distribution $F(x, Q^2)$ above, the convolution $\otimes$ is defined as,

$$P_{ij} \otimes F_j \equiv \int_x^1 \frac{dy}{y} P_{ij}\left(\frac{x}{y}, Q^2\right) F_j(y, Q^2) \tag{2.3}$$

where $P_{ij}(x, Q^2)$ consists of the splitting functions $p_{ij}^{(0)}$ in Leading Order (LO) and $p_{ij}^{(1)}$ in next to leading order (NLO),

$$P_{ij} = \left[\frac{\alpha_s}{2\pi}\right] p_{ij}^{(0)} + \left[\frac{\alpha_s}{2\pi}\right]^2 p_{ij}^{(1)} + \cdots \tag{2.4}$$

The main difference between the evolution of the photon structure function and that of the proton structure function is the presence of the inhomogeneous K_i terms in the evolution equations. Essentially these consist of $\gamma \to$ quark and $\gamma \to$ gluon splitting functions. They appear in the evolution equations without any convolution with a parton distribution since the "photon density" inside a photon is taken to be $\delta(1-x)$ up to corrections of order α_{em}.

$$K_i = \left[\frac{\alpha_{em}}{2\pi}\right] k_i^{(0)} + \left[\frac{\alpha_{em}}{2\pi}\right]\left[\frac{\alpha_s}{2\pi}\right] k_i^{(1)} + \cdots \tag{2.5}$$

F_2^γ in (LO) is given by,

$$\frac{1}{x} F_2^\gamma = \{q_{\rm NS} + \langle e^2 \rangle \Sigma\} \tag{2.6}$$

where

$$q_{\rm NS} = \sum_f \left(e_q^2 - \langle e^2 \rangle\right)(q_i + \bar{q}_i) \quad , \quad \langle e^k \rangle = \frac{1}{f}\sum_f e_q^k$$

$\alpha_s(Q^2)$ evolves according to

$$\frac{\alpha_s(Q^2)}{4\pi} = \frac{1}{\beta_0 \ln Q^2/\Lambda^2} - \frac{\beta_1}{\beta_0^3} \frac{\ln(\ln Q^2/\Lambda^2)}{(\ln Q^2/\Lambda^2)^2} \tag{2.7}$$

where $\beta_0 = 11 - 2f/3$ and $\beta_1 = 102 - 38f/3$. All expressions refer to the $\overline{MS}$ renormalization scheme hence we use $\Lambda_{\overline{MS}}$ which depends f. We evolve in (NLO) to the t-quark threshold and then we evolve in (LO) thereafter by setting β_1 to zero. This is because we can only evolve in (LO) above the SUSY threshold so in order to compare F_2^γ for different values of the squark mass M_s we must evolve in the same way from the t-quark threshold. Quark masses are taken as $M(c) = 1.5\,GeV$, $M(b) = 4.5\,GeV$, $M(t) = 174\,GeV$.

Our input data were parameterizations [3] of virtual photon parton densities taken at the c-quark threshold. A c-quark mass of $1.5\,GeV$ limits P^2 to less than $1.8\,GeV$ which gives a small ratio $r = P^2/Q^2 \simeq 10^{-6}$ at high Q^2. We could not find reliable parameterizations at higher Q^2 that were dependent on P^2.

In order to evolve to the SUSY threshold we use (LO) and (NLO) splitting functions [4] in (2.4) and inhomogeneous terms [2] in (2.5).

Above the SUSY threshold M_s we are also concerned with squark and gluino distributions. As before we have a nonsinglet (S_l) and singlet (Γ) sector,

$$\begin{aligned} S_3 &= 4(s_u - s_d) \\ S_8 &= 4(s_u + s_d - 2s_s) \\ S_{15} &= 4(s_u + s_d + s_s - 3s_c) \\ S_{24} &= 4(s_u + s_d + s_s + s_c - 4s_b) \\ S_{35} &= 4(s_u + s_d + s_s + s_c + s_b - 5s_t) \\ \Gamma &= 4\sum_i^f s_i \end{aligned}$$

where s_u, s_d, s_s, s_c, s_b and s_t refer to the squark distributions. The factor of 4 arises because $s_i^R = s_i^L = \bar{s}_i^R = \bar{s}_i^L$.

For simplicity all squark distributions start at zero at the SUSY threshold corresponding to a common squark mass M_s, although we could introduce them in steps as before with the quarks. The gluino distribution starts at zero at the gluino threshold corresponding to a gluino mass M_g.

The evolution is controlled by the following inhomogeneous DGLAP differential equations. Each set of nonsinglets are coupled ie. T_3 with S_3, T_8 with S_8, etc...

$$\frac{dT_l}{d\ln Q^2} = P_{TT}\otimes T_l + P_{TS}\otimes S_l + K_T$$

$$\frac{dS_l}{d\ln Q^2} = P_{ST}\otimes T_l + P_{SS}\otimes S_l + K_S \tag{2.8}$$

Given that in general the gluino mass M_g is greater than the squark mass M_s the nonsinglet sector evolution is given in the region $4M_g^2 > Q^2 \geq 4M_s^2$ by,

$$\frac{d\Sigma}{d\ln Q^2} = P_{\Sigma\Sigma}\otimes\Sigma + P_{\Sigma G}\otimes G + P_{\Sigma\Gamma}\otimes\Gamma + K_\Sigma$$

$$\frac{dG}{d\ln Q^2} = P_{G\Sigma}\otimes\Sigma + P_{GG}\otimes G + P_{G\Gamma}\otimes\Gamma + K_G$$

$$\frac{d\Gamma}{d\ln Q^2} = P_{\Gamma\Sigma}\otimes\Sigma + P_{\Gamma G}\otimes G + P_{\Gamma\Gamma}\otimes\Gamma + K_\Gamma \tag{2.9}$$

and in the region $Q^2 \geq 4M_g^2$ by,

$$\frac{d\Sigma}{d\ln Q^2} = P_{\Sigma\Sigma}\otimes\Sigma + P_{\Sigma G}\otimes G + P_{\Sigma\Gamma}\otimes\Gamma + P_{\Sigma L}\otimes L + K_\Sigma$$

$$\frac{dG}{d\ln Q^2} = P_{G\Sigma}\otimes\Sigma + P_{GG}\otimes G + P_{G\Gamma}\otimes\Gamma + P_{GL}\otimes L + K_G$$

$$\frac{d\Gamma}{d\ln Q^2} = P_{\Gamma\Sigma}\otimes\Sigma + P_{\Gamma G}\otimes G + P_{\Gamma\Gamma}\otimes\Gamma + P_{\Gamma L}\otimes L + K_\Gamma$$

$$\frac{dL}{d\ln Q^2} = P_{L\Sigma}\otimes\Sigma + P_{LG}\otimes G + P_{L\Gamma}\otimes\Gamma + P_{LL}\otimes L + K_L \tag{2.10}$$

where L is the gluino distribution. In the limit where the gluino mass M_g is taken to be equal to the squark mass M_s we do not need (2.9).

The $P_{ij}(x,Q^2)$ are now a different SUSY set of (LO) splitting functions [5]. In analogy with the K_i used below the SUSY threshold, K_S and K_Γ correspond to the photon to squark splitting functions and K_L corresponds to the photon to gluino splitting functions. We also account for the fact that the running of the strong coupling is now controlled by the SUSY $\beta-$function [6], i.e. we take $\beta_0 = 9 - f/2$ and $\beta_1 = 54 - 17f/3$.

The tree level squark contribution to F_2^γ (Appendix A) is important in determining the $\gamma \to$ squark splitting function i.e., the squark inhomogeneous terms in (2.8), (2.9) and (2.10). K_S and K_Γ, apart from their respective coefficients, are essentially the same in (LO) and are obtained from the tree level contribution to F_2^γ for squark production which is given in (A.1). We have obtained them in the same way that the photon to quark splitting functions are obtained from the tree level contribution to F_2^γ for quark production,

$$k_S^{(0)}(x) = 2\times 3f\,\left(\langle e^4\rangle - \langle e^2\rangle^2\right)\,2\,\left(2x(1-x)\right) \tag{2.11}$$

$$k_\Gamma^{(0)}(x) = 2\times 3f\,\langle e^2\rangle\,2\,\left(2x(1-x)\right) \tag{2.12}$$

where the factor of $2\times$ accounts for left and right handed squarks. The other coefficients are standard. The $(2x(1-x))$ accounts for the linear $\ln Q^2$ evolution of the distributions in (LO), which is valid up to $(\ln(Q^2/4M_s^2))^2$. As with the gluon splitting function the gluino splitting function is zero in (LO),

$$k_L^{(0)} = 0 \tag{2.13}$$

The full threshold condition for the production of squarks is given by,

$$Q^2\frac{(1-x-r\,x)}{x} \geq 4M_s^2 \tag{2.14}$$

which importantly depends both on x and Q^2. We attempt to apply this condition above $Q^2 = 4M_s^2$. At some Q^2 there will be a value of x, say $x_s(Q^2)$, above which squarks cannot be produced due to the this condition. A number of convolutions, (2.3), must be performed in order to calculate the change in each of the distributions as we evolve in $\ln Q^2$. Any particular convolution evaluated at x is an integral in a dummy variable y in the region $y \geq x$. Since all convolutions are done numerically SUSY splitting functions can be used for $y < x_s$, where squarks can be produced and standard splitting functions can be used for $y > x_s$, where squarks cannot be produced. The running of the strong coupling is affected by the existence of SUSY particles. We allow for this by imposing continuity of the coupling through the simplistic squark threshold at $Q^2 = 4M_s^2$ and using $\beta_0 = 11 - 2/3f_q - 2 - 1/3f_s$. Essentially this means that β_0 in (2.7) changes from 7 to 3 above $Q^2 = 4M_s^2$. However we impose the condition in (2.14) in order to choose whether or not to use the SUSY altered coupling to calculate the change in each distribution at a particular x and Q^2. This means that for any $Q^2 \geq 4M_s^2$, F_2^γ with SUSY contributions will coincide with F_2^γ without SUSY contributions for $x > x_s$. F_2^γ is unaltered by SUSY contributions in the region $x \geq x_s(Q^2)$, as would be expected. This is not an ideal strategy for incorporating the threshold condition in (2.14) since for $Q^2 < 4M_s^2$ there will always be an $x < x_s(Q^2)$ such that squarks can be produced. However it is an improvement on just using $Q^2 \geq 4M_s^2$ as a squark threshold condition.

The contribution to F_2^γ from squark production, (A.1), is obtained from ordinary perturbation theory and includes a term proportional to $\ln(Q^2/4M_s^2)$, which is already accounted for as K_S or K_Γ. In order to approximate the correct threshold behaviour we try to isolate the part of (A.1) that is not used in the squark splitting functions and introduce this contribution as B_γ^{sq} below.

Above the SUSY threshold F_2^γ is then obtained from

$$\frac{1}{x}F_2^\gamma = \left\{q_{\rm NS} + \langle e^2\rangle\,\Sigma\right\} + \left\{Sq_{\rm NS} + \langle e^2\rangle\,\Gamma\right\} + 2\times 3f\langle e^4\rangle\left[\frac{\alpha_{em}}{4\pi}\right]B_\gamma^{sq} \tag{2.15}$$

where the factor of 2× accounts for left and right handed squarks. The B_γ^{sq} term is also subject to the threshold condition in (2.14), meaning that it is not included in the region $x > x_s$.

Essentially what we do is to take the squark contribution to F_2^γ and subtract off the $(2x(1-x))$ part that accounted for the $\ln Q^2$ dependence in $k_S^{(0)}$ and $k_\Gamma^{(0)}$. Given that the coefficient of B_γ^{sq} is $\left(2 \times 3f\langle e^4\rangle\,\alpha_{em}/4\pi\right)$ in (2.15),

$$B_\gamma^{sq} \;=\; 4\left\{\left(\frac{1}{3\,e_{sq}^4\,\frac{\alpha_{em}}{\pi}\,x}\right) \times F_{2,sq}^\gamma - 2x(1-x)\ln\left(\frac{Q^2}{4M_s^2}\right)\right\} \tag{2.16}$$

where $F_{2,sq}^\gamma$ is given in A.1, will give the correct contribution.

We note that the difference between using (A.1) and (A.2) is negligible in our case because we are limited to $P^2 < 1.8\,GeV^2$ at the c-quark threshold, giving an $r \simeq 10^{-6}$ above the SUSY threshold.

However this is a different way of treating the threshold behaviour from that in [2]. At $Q^2 \gg 4M_s^2$ it satisfies the Renormalization Group equations since the dominant part is in the inhomogeneous term. In the region $Q^2 \simeq 4M_s^2$ this approach will reproduce the perturbative calculation with the correct threshold behaviour up to $(\ln(Q^2/4M_s^2))^2$. There should of course be a small mismatch at large Q^2 and large x. However we have eradicated this by incorporating the threshold condition from (2.14) into both the splitting functions and the B_γ^{sq} term as explained above. Obviously (A.1) exhibits a functional dependence on Q^2 that is more complicated than just $\ln Q^2$. This means that our choice of inhomogeneous squark terms do not fully describe the Q^2 evolution of F_2^γ and that our B_γ^{sq} term is also therefore an approximation. This can result in a discontinuity in the region $x_s(Q^2)$ which we will show in our results in the next section.

We have not invoked the full theoretical framework for dealing with heavy flavour contributions to the structure function as decribed in [9] and [10] for example. This is primarily because we are concerned with whether supersymmetry exhibits a measurable effect on the structure function rather than exact numerical predictions in the threshold region.

Finally it should be noted how quickly F_2^γ changes away from the threshold with decreasing x. In (A.1), the term

$$v = [1 - 4M_s^2 x/Q^2(1-x)]^{1/2}$$

moves rapidly away from zero in decreasing x meaning that the coefficients of

$$\ln\left(\frac{1+v}{1-v}\right) \quad \text{and} \quad v$$

in (A.1) give rise to a real threshold contained in the B_γ^{sq} term.

To summarize, we take parameterizations of quark and gluon distributions inside a virtual photon at the c-threshold. Using DGLAP inhomogeneous differential equations we evolve the relevant non-singlet, singlet and gluon distributions up to the SUSY threshold. From here we run the distributions separately, including or not, the effects of squarks and gluinos. At some $\sqrt{Q^2}$ we form F_2^γ for the virtual photon in such a way as to take account of the SUSY threshold condition.

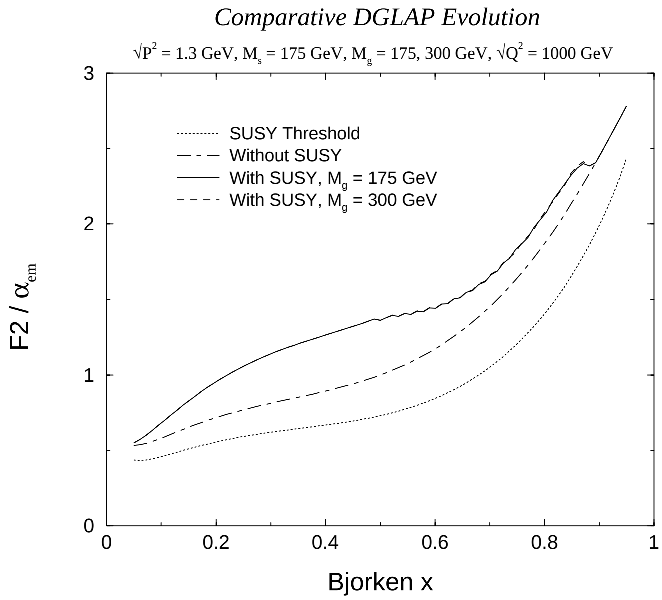

Fig. 1. Comparative Evolution of Structure Function with and without SUSY splitting functions. Difference due to a higher gluino mass M_g is negligible

3 Results

The variable parameters of the evolution are the P^2 (target virtuality), M_s (squark mass), M_g (gluino mass), Q^2 (incident virtuality) and Bjorken x. We took these in the ranges,

$$0 \;\le\; \sqrt{P^2} \;\le\; 1300\,MeV$$
$$175\,GeV \;\le\; M_s \;\le\; 300\,GeV$$
$$175\,GeV \;\le\; M_g \;\le\; 300\,GeV$$
$$700\,GeV \;\le\; \sqrt{Q^2} \;\le 1500\,GeV$$

and in all cases F_2^γ/α_{em} is actually plotted.

Figure 1 shows a generalised evolution to $1000\,GeV$. The bottom graph corresponds to F_2^γ evaluated at the SUSY threshold $\sqrt{Q^2} = 2M_s = 350\,GeV$. This serves as a base reference since it is at this point that the SUSY effects are included in the evolution. There is a considerable difference to F_2^γ on including supersymmetric effects. We agree with the general conclusions made in [7] that F_2^γ with SUSY contributions is flatter and strongly increases for decreasing values of x. Note that allowing the gluino mass to be greater than the squark mass produces a negligible effect. Note also that the graphs coincide above the

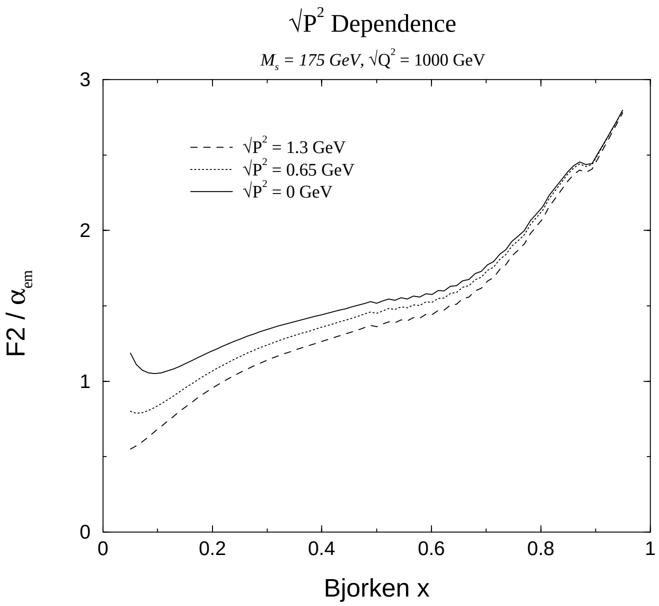

Fig. 2. $\sqrt{P^2}$ dependence of structure function for fixed squark mass M_s at a fixed probe virtuality $\sqrt{Q^2}$

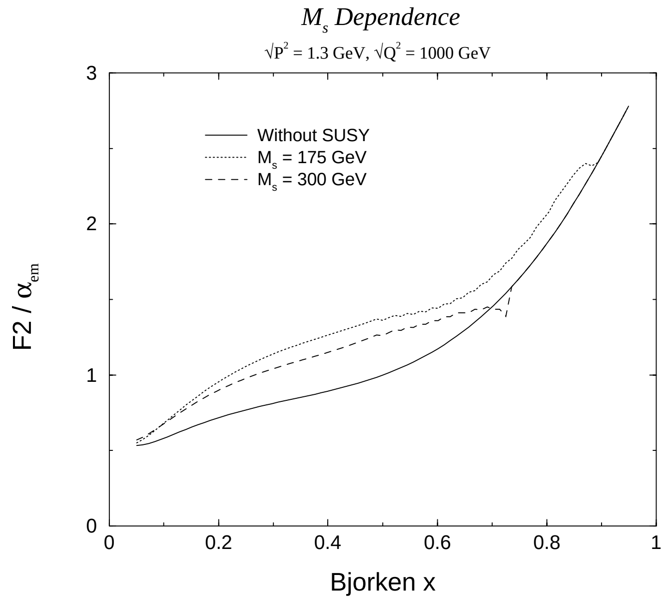

Fig. 3. Dependence of structure function on squark mass M_s at a fixed target virtuality $\sqrt{P^2}$ and probe virtuality $\sqrt{Q^2}$

squark threshold $x_s(Q^2)$ given in (2.14), this being due to it being incorporated into the splitting functions and the B_γ^{sq} term, as described in the previous section.

From here on we plot $M_g = M_s$ since we have shown the $M_g > M_s$ difference to be negligible.

Figure 2 shows P^2 dependence up to $1300\,MeV$. We are limited by our parameterizations in that they are restricted in P^2 at the c-quark threshold. However non-negligible differences can be noted in low x even at $\sqrt{P^2} = 1300\,MeV$.

Figure 3 shows M_s dependence between $175\,GeV$ and $300\,GeV$. The lowest graph is without the SUSY contributions. As the squark mass M_s increases, F_2^γ approaches the non-SUSY limit as if the non-SUSY graph corresponds to exciting squarks of infinite mass. Also the thresholds move to lower x as the threshold condition (2.14) requires higher Q^2 to produce squarks of higher mass. We can see that for $M_s = 300\,GeV$ there is a discontinuity around $x_s = 0.74$. This is due to the fact that our treatment of the squark threshold using the B_γ^{sq} term in (2.15) is only approximate as discussed in the previous section. The discontinuity is more apparent for $M_s = 300\,GeV$ than for $M_s = 175\,GeV$ since the error increases as the ratio $4M_s^2/Q^2$ approaches unity.

Figures 4 and 5 show how F_2^γ varies with $\sqrt{Q^2}$ at two fixed values of x. All graphs show how the distributions must approach the non-SUSY distribution for high M_s. However for $x = 0.66$ we can see the gradual approach to a threshold in increasing M_s. For $M_s = 275\,GeV$ it is evident that for low Q^2 squarks cannot be produced and the distribution coincides with the non-SUSY distribution. Then apart from the discontinuity mentioned previously the distribution rises in higher Q^2.

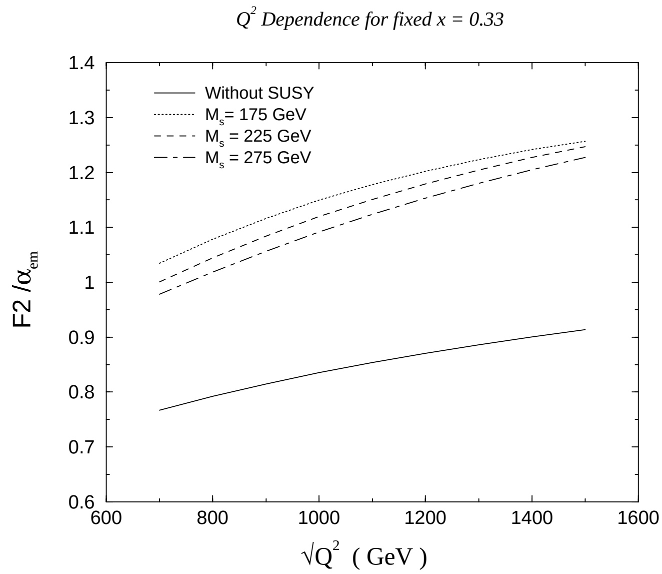

Fig. 4. $x = 0.33$

4 Conclusions

We see from Figs. 1-5 that if one can build a machine for which values of Q^2 approach 1 TeV^2 (about twice the squark production threshold) there is a substantial increase in the value of F_2^γ for the photon. Indeed, the evolution between the SUSY threshold and 1 TeV is more than doubled if SUSY particles, taken to have a mass of 175 GeV, are present. The difference between the structure functions with and without SUSY in the middle range of Bjorken-x is over 30%, which should be easily detectable.

The effect at $Q^2 = 1$ TeV^2 is, of course, diminished if the SUSY threshold is increased. However, we note that taking the squark masses to 300 GeV only has a small

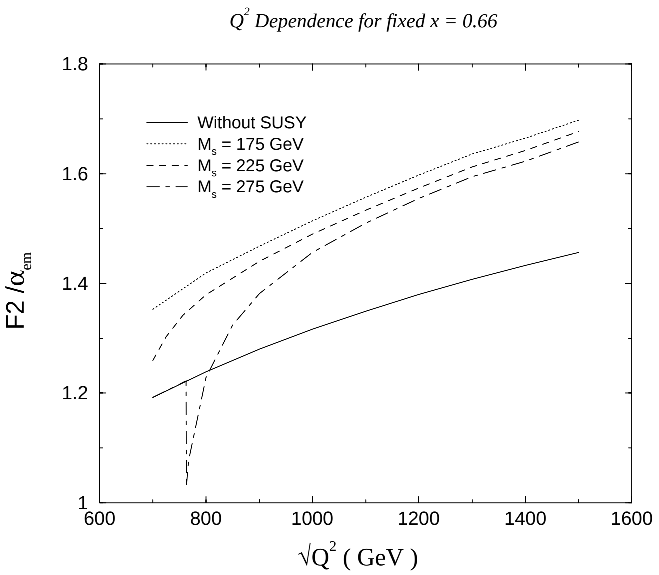

Fig. 5. $x = 0.66$

effect on F_2^γ. Conversely, if the squark masses turn out to be substantially lighter than 175 GeV[1], (which is not currently ruled out), there would be a significant effect on the structure functions at values of Q^2 significantly below 1 TeV^2.

The effect also diminishes if the target photon is off-shell, as will usually be the case. However, we see from Fig. 2 that this effect is modest.

The results are fairly insensitive to the mass of the gluino. This is not surprising as the gluino contributes very indirectly - it can only be produced by a secondary emission from the target photon and then only probed through a further interaction with squarks. Taking the mass of the gluino below that of the squark, would have had a negligible effect as it is clear that it is the squark threshold and not the gluino threshold that must be crossed before there is any effect on the photon structure function.

In summary, we see that the effect of SUSY on the photon structure function provides a further good reason for designing a large electron-positron collider that would be capable of reaching values of Q^2 above the SUSY threshold for the middle range of Bjorken-x.

Appendix A

The contribution to F_2^γ of a left or right handed squark in deep inelastic scattering on a photon has been calculated [7],

$$F_{2,sq}^\gamma = 3e_{sq}^4 \frac{\alpha}{\pi} x \left\{ \left[2x(1-x) + \tau x(3x-1) + \frac{1}{2}\tau^2 x^2 \right] \times \ln\left(\frac{1+v}{1-v}\right) + [1 - 8x(1-x) + \tau x(1-x)]\, v \right\} \quad (A.1)$$

where,

$$\tau = 4M_s^2/Q^2$$

$$v = [1 - \tau x/(1-x)]^{1/2}$$

and e_{sq} is the squark charge in units of e.

We calculated this expression for the case $P^2 \neq 0$, where $r = P^2/Q^2$. The above relation is recovered on $r \to 0$ with,

$$\begin{aligned} F_{2,sq}^\gamma = 3e_{sq}^4 \frac{\alpha}{\pi} x \Big\{ & B(M_s^2/Q^2)^2(1/FG)(16x^2) \\ & + B(M_s^2/Q^2)(1/FG) \\ & \times(-48x^4r^2 + 48x^3r + 4x^2r - 8x^2) \\ & + B(1/FG)(-12x^4r^3 + 12x^3r^2 - 2x^2r) \\ & + B(1 - 6x^2r + 6x^2 - 6x) \\ & + \ln(F/G)(M_s^2/Q^2)^2(B/\eta)(8x^2/b) \\ & + \ln(F/G)(M_s^2/Q^2)(B/\eta) \\ & \times(24x^4r^2/b + 2x^2r/b + 12x^2 - 2x/b - 2x) \\ & + \ln(F/G)(B/\eta)\Big(\frac{1}{2} + 6x^4r^3/b + 12x^4r^2/b \\ & - 12x^3r^2/b - 12x^3r/b + 11x^2r/b - 3x^2r \\ & + 4x^2/b - 6x^2 + \frac{1}{2}xr/b \\ & - \frac{1}{2}xr - 3x/b + 5x - \frac{1}{2b}\Big) \Big\} \end{aligned} \quad (A.2)$$

where,

$$\begin{aligned} b &= 1 - 2xr \\ F &= 1 + \eta(1 - 2xr) \\ G &= 1 - \eta(1 - 2xr) \\ B &= \sqrt{1 - \frac{4M_s^2 x}{Q^2(1 - x - xr)}} \\ \eta &= \frac{B}{b}\sqrt{1 - 4x^2 r} \end{aligned}$$

These equations are important in determining the $\gamma \to$ squark splitting function i.e., the squark inhomogeneous K_S and K_Γ terms. Also in determining the extra B_γ^{sq} term in (2.15). The choice is dependent on the squark threshold condition (2.14), which is function of both x and Q^2. At a particular Q^2 there will be a region $x > x_s(Q^2)$ where squarks cannot be produced.

[1] The lowest value we take for the squark mass is 175 GeV since this is above the threshold for t-quark and we find it useful to make comparisons of the evolution of the structure function in the presence of SUSY with that without SUSY but with six active flavours

References

1. E.Witten, Nucl. Phys. **B120**, 189 (1977)
2. M. Glück, E. Reya, Phys. Rev. **D28**, 2749 (1983); M. Scott, W.J. Stirling, Phys. Rev. **D30**, 157 (1984); M. Drees M. Glück, E. Reya, Phys. Rev. **D30**, 2316 (1984)
3. M. Glück, E. Reya, M. Stratmann, Phys. Rev. **D54**, 5515 (1996)
4. R. Ellis, W. Stirling, B. Webber, QCD and Collider Physics. pg 109-113 Cambridge University Press, 1996
5. C. Kounnas, D. Ross, Nucl. Phys. **B214**, 317 (1983); S.K. Jones, C.H. LLewellyn-Smith, Nucl. Phys. **B217**, 145 (1983)
6. D.R.T. Jones, Nucl. Phys. **B87**, 127 (1975)
7. E. Reya, Phys. Lett. **B124**, 424 (1983)
8. M. Glück, E. Reya, A. Vogt, Phys. Rev. **D45**, 3986 (1991)
9. R. Demina et al., **hep-ph/0005112** (2000)
10. M. Buza et.al, Nucl. Phys. **B472** 611 (1996); M. Buza, V. Matiouine, J. Smith, W.L. van Neerven, Eur. Phys. J. **C1** 301 (1998); R.S.Thorne, R.G. Roberts, Phys. Rev. **D57**, 6781 (1998); R.S.Thorne, R.G. Roberts, Phys. Lett. **B421** 303 (1998); A. Martin, R. Roberts, W. Stirling, R. Thorne, Eur. Phys. J. **C4** 463 (1998)